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BOUNDARY VALUE PROBLEMS OF THE KINETIC THEORY OF GASES AND IRREVERSIBLE THERMODYNAMICS

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The classic formula for integral entropy production is substantiated by kinetic considerations on the example of gas flow in a plane — parallel channel under conditions in which the mean free path is comparable with the distance between walls. A system of moment equations for calculating coefficients of expansion of the function in velocity polynomials is derived on the basis of local entropy production. The obtained system is solved for the flow of a simple gas, induced by longitudinal pressure and temperature gradients. It is shown by direct calculations that the approximate solution obtained in this manner results in the exact fulfilment of the Onsager symmetry relations for kinetic coefficients.

The Chapman-Enskog method, which is applicable in cases when the state of gas differs only slightly from equilibrium (e.g., when gradients of macroscopic quantities are not large), is widely used for obtaining a closed system of transport equations, i.e. equations of hydrodynamics. The last condition reduces to the stipulation of smallness of the Knudsen number. At fairly high vacuums the Chapman-Enskog method in its classical form becomes inapplicable.

Problems with two characteristic geometric dimensions are frequent. It is possible to construct in that case a generalized Chapman-Enskog method with two Knudsen numbers, one of which is small and the other, arbitrary. The physically obvious stipulation is that in case of smallness of the two Knudsen numbers the generalized method converts to the classic Chapman-Enskog method.

1. We write the Boltzmann equations as

$$\mathbf{v}_\alpha \frac{\partial f_\alpha}{\partial \mathbf{r}} = I(f_\alpha, f_\alpha) \tag{1.1}$$

where $I(f_\alpha, f_\alpha)$ is the Boltzmann collision integral, and seek a solution of Eq. (1.1) of the form

$$f_\alpha = f_\alpha^\circ (1 + \varphi_\alpha)$$

where φ_α is a small addition proportional to the "longitudinal" Knudsen number (below for brevity we shall call "longitudinal" and "transverse" Knudsen numbers the ratios $\mathrm{Kn}_1 = \lambda / S$ and $\mathrm{Kn}_2 = \lambda / L$, respectively, with S and L being characteristic geometric scales of the problem and λ representing the mean free path of gas molecules).

After linearization we obtain the equation

$$v_{\alpha x}\frac{\partial f_\alpha^\circ}{\partial x}+v_{\alpha z}\frac{\partial f_\alpha^\circ}{\partial z}+v_{\alpha x}\varphi_\alpha\frac{\partial f_\alpha^\circ}{\partial x}+f_\alpha^\circ v_{\alpha x}\frac{\partial\varphi_\alpha}{\partial x}=I(f_\alpha^\circ,f_\alpha^\circ)+f_\alpha^\circ I(\varphi_\alpha) \tag{1.2}$$

where z and x are the longitudinal and transverse coordinates. The sequence of consecutive approximations (with respect to Kn_1) is of the form

$$v_{\alpha x}\frac{\partial f_\alpha^\circ}{\partial x}=I(f_\alpha^\circ,f_\alpha^\circ) \tag{1.3}$$

$$v_{\alpha z}\frac{\partial f_\alpha^\circ}{\partial z}+f_\alpha^\circ v_{\alpha x}\frac{\partial\varphi_\alpha}{\partial x}+v_{\alpha x}\varphi_\alpha\frac{\partial f_\alpha^\circ}{\partial x}=f_\alpha^\circ I(\varphi_\alpha) \tag{1.4}$$

The local Maxwell distribution whose parameters are independent of x satisfy Eq. (1.3). We use it as the zero approximation of the sought solution. The solution of Eq. (1.4) is valid for any arbitrary Knudsen number, and makes it possible to determine the macro-properties of flow for arbitrary vacuums at any distance from the surfaces bounding the gas. An exact solution of Eq. (1.4) is only possible in certain special cases [1]. Hence it is usual to resort to various approximate methods when considering such problems. Here we use the moment method.

2. First, we shall show that thermodynamics of irreversible processes can be successfully applied to the study of phenomena in which properties of gas in Knudsen layers at the gas-solid body interface play a substantial part.

We use Eq. (1.4) for investigating the gas flow in a plane-parallel channel of length S and width $2L$. Multiplying both sides of the equation by φ_α and integrating with respect to momenta $\mathbf{p}_\alpha$ of molecules, we obtain

$$\int\varphi_\alpha v_{\alpha z}f_\alpha^\circ\frac{\partial\ln f_\alpha^\circ}{\partial z}\,d\mathbf{p}_\alpha+\int\varphi_\alpha v_{\alpha x}f_\alpha^\circ\frac{\partial\varphi_\alpha}{\partial x}\,d\mathbf{p}_\alpha=\int\varphi_\alpha f_\alpha^\circ I(\varphi_\alpha)\,d\mathbf{p}_\alpha \tag{2.1}$$

We multiply both sides of Eq. (2.1) by the Boltzmann constant, sum up over all kinds of α molecules, and average over the channel volume. Denoting the entropy production per unit of gas volume in the channel by $\Delta\sigma_t$, we obtain

$$\sum_\alpha\frac{k}{2LS}\int_0^S\int_{-L}^{L}\left(\varphi_\alpha v_{\alpha z}\frac{\partial\ln f_\alpha^\circ}{\partial z}+v_{\alpha x}\frac{1}{2}\frac{\partial\varphi_\alpha^2}{\partial x}\right)f_\alpha^\circ d\mathbf{p}_\alpha dz\,dx=-\Delta\sigma_t \tag{2.2}$$

We substitute for f_α° the expression

$$\exp\left[\frac{m_\alpha}{kT}\left(\mu_\alpha-\frac{1}{2}v_\alpha^2\right)\right]$$

where μ_α is the chemical potential of component α and m_α is the mass of a molecule, and introduce the notation $\mathbf{J}_\alpha^m(x)$ and $\mathbf{J}_\alpha^u(x)$ for the density vectors of mass and energy fluxes, respectively. Then instead of (2.2) we have

$$\frac{1}{2LS}\sum_\alpha\left\{\int_0^S\int_{-L}^{L}\left[J_{\alpha z}^m(x)\frac{\partial}{\partial z}\left(\frac{\mu_\alpha}{T}\right)-J_{\alpha z}^u(x)\frac{\partial}{\partial z}\left(\frac{1}{T}\right)\right]dz\,dx\right.$$

$$+ kS\int \int\limits_{-L}^{L} f_\alpha^\circ v_{\alpha x} \frac{\partial}{\partial x}\left(\frac{\varphi_\alpha^2}{2}\right) d\mathbf{p}_\alpha dx\Big\} = - \Delta\sigma_t$$

$$\mathbf{J}_\alpha^m(x) = m_\alpha \int \varphi_\alpha f_\alpha^\circ \mathbf{v}_\alpha d\mathbf{p}_\alpha$$

$$\mathbf{J}_\alpha^u(x) = \frac{1}{2} m_\alpha \int \varphi_\alpha f_\alpha^\circ \mathbf{v}_\alpha v_\alpha^2 d\mathbf{p}_\alpha$$

Denoting by $\overline{\mathbf{J}_\alpha^m}$ and $\overline{\mathbf{J}_\alpha^u}$ the fluxes of mass and energy averaged over the channel cross section, we finally obtain

$$\Delta\sigma_t = \sum_\alpha \left[(\bar{\mathbf{J}}_\alpha^u - \mu_\alpha \bar{\mathbf{J}}^m)\frac{\nabla T}{T^2} + \bar{\mathbf{J}}^m \frac{\nabla \mu_\alpha}{T}\right] \tag{2.3}$$

$$- \frac{k}{2L} \sum_\alpha \int f_\alpha^\circ v_{\alpha x} \frac{\varphi_\alpha^2}{2} d\mathbf{p}_\alpha \Big|_{-L}^{L}, \quad \nabla T = \frac{T(S) - T(0)}{S}$$

$$\Delta\mu_\alpha = \frac{\mu_\alpha(S) - \mu_\alpha(0)}{S}$$

Thus the formula for entropy production in gas differs from the classic formula [3] by the additional term related to the properties of gas in the Knudsen layer at the channel wall.

Formula (2.3) determines the entropy production that is due only to collisions between the molecules themselves. In the considered case the interaction between gas and channel walls plays an important part. It also defines a certain amount of entropy production in the gas, which must be added to the right-hand side of formula (2.3). It can be calculated as follows. The distribution of molecules impinging on the wall surface at $x = L$ is f_α^+ and of molecules scattered by the latter is f_α^- (the superscript defines the sign of the x-component of molecule velocity). If the law of interaction between gas and wall is not specular, these functions are substantially different. Thus the molecules impinging on the wall carry an entropy flux of density $\mathbf{J}^s(0)$ and those reflected from the wall, a flux of density $\mathbf{J}^s(\delta)$

$$\mathbf{J}^s(0) = -k \sum_\alpha \int\limits_{v_x > 0} \mathbf{v} f_\alpha^+ \ln f_\alpha^+ d\mathbf{p}_\alpha$$

$$\mathbf{J}^s(\delta) = -k \sum_\alpha \int\limits_{v_x < 0} \mathbf{v} f_\alpha^- \ln f_\alpha^- d\mathbf{p}_\alpha$$

where δ is the nominal thickness of the wall material in which the described transformation of the distribution function takes place. The equation of entropy balance is

$$\operatorname{div} \mathbf{J}^s = \Delta\sigma_w^*$$

where $\Delta\sigma_w^*$ is the entropy production in the δ layer per unit of the layer volume. We integrate this equality over the layer volume $S\delta$ and relate it to the channel volume. For the entropy production in the surface layer of the wall $x = L$ we obtain the formula

$$\Delta\sigma_w = \frac{k}{2L} \sum_\alpha \Big\{ \int\limits_{v_x < 0} v_{\alpha x} f_\alpha^- \ln f_\alpha^- d\mathbf{p}_\alpha + \int\limits_{v_x > 0} v_{\alpha x} f_\alpha^+ \ln f_\alpha^+ d\mathbf{p}_\alpha \Big\}$$

$$= \frac{k}{2L} \sum_\alpha \int v_{\alpha x} f_\alpha \ln f_\alpha d\mathbf{p}_\alpha$$

The formula for entropy production in the surface layer of the wall $x = -L$ is of the same form. Taking into account the difference of signs of velocity components v_x, we finally obtain for the entropy production by the collision of gas molecules with the channel walls the formula

$$\Delta\sigma_w = \frac{k}{2L}\sum_\alpha \int v_{\alpha x} f_\alpha \ln f_\alpha \, d\mathbf{p}_\alpha \Big|_{-L}^{L} = \frac{k}{2L}\sum_\alpha \int v_{\alpha x} f_\alpha{}^{\circ} \frac{\varphi_\alpha{}^2}{2} d\mathbf{p}_\alpha \Big|_{-L}^{L} \qquad (2.4)$$

The total entropy production per unit of channel volume $\Delta\sigma = \Delta\sigma_t + \Delta\sigma_w$ has the classic form of the bilinear combination of thermodynamic forces and fluxes. This result positively solves the question of validity of application of irreversible process thermodynamics to problem in which the properties of gas inside Knudsen layers are effectively taken into consideration. In particular the thermodynamic equations of motion retain their form. Thus for a simple gas we have

$$\bar{\mathbf{q}} = L_{11}\frac{\nabla T}{T^2} + L_{12}\frac{\nabla p}{T} \qquad (2.5)$$

$$\bar{\mathbf{u}} = \frac{\mathbf{J}^m}{\rho} = L_{21}\frac{\nabla T}{T^2} + L_{22}\frac{\nabla p}{T}$$

and for a binary gas mixture

$$\bar{\mathbf{q}} = \alpha_{11}\frac{\nabla T}{T^2} + \alpha_{12}\frac{\nabla c_1}{T} + \alpha_{13}\frac{\nabla p}{T} \qquad (2.6)$$

$$\overline{(\mathbf{u}_1 - \mathbf{u}_2)}\, p = \alpha_{21}\frac{\nabla T}{T^2} + \alpha_{22}\frac{\nabla c_1}{T} + \alpha_{23}\frac{\nabla p}{T}$$

$$\bar{\mathbf{v}}_0 + \frac{m_2 - m_1}{\rho}\frac{n_1 n_2}{n}\overline{(\mathbf{u}_1 - \mathbf{u}_2)} = \alpha_{31}\frac{\nabla T}{T^2} + \alpha_{32}\frac{\nabla c_1}{T} + \alpha_{33}\frac{\nabla p}{T}$$

$$\mathbf{q} = \sum_\alpha \left(\mathbf{J}_\alpha{}^u - \frac{5}{2}\frac{kT}{m_\alpha}\mathbf{J}_\alpha{}^m\right), \quad \mathbf{v}_0 = \frac{1}{\rho}\sum \rho_\alpha \mathbf{u}_\alpha$$

$$\mathbf{u}_\alpha = \frac{\mathbf{J}_\alpha{}^m}{\rho_\alpha}, \quad c_\alpha = \frac{n_\alpha}{n}$$

where $\mathbf{q}$ is the vector of heat flux density, $\mathbf{u}_\alpha$ is the mass velocity of molecules of the α kind, $\mathbf{v}_0$ is the mean mass velocity of the mixture, ρ is the density, p is the pressure, c_α is the concentration of particles of the α kind, and n is the total number of particles in a unit of volume. Note that the combination

$$\mathbf{v}_0 + \frac{m_2 - m_1}{\rho}\frac{n_1 n_2}{n}(\mathbf{u}_1 - \mathbf{u}_2) = \frac{1}{n}\sum_\alpha n_\alpha \mathbf{u}_\alpha$$

constitutes the mean molar velocity of the mixture.

3. The determination of moments of the kinetic equation poses the question of selecting velocity polynomials that are to be used for this purpose. So far this problem has not been satisfactorily elucidated in literature. It is, therefore, pertinent to propose one of the criteria of such selection. We shall base it on the principles of thermodynamics of irreversible processes.

In a stationary case the equation of entropy balance in gas is

$$\operatorname{div} \mathbf{J}^s = \Delta\sigma \qquad (3.1)$$

It can be shown that equality (3.1) is equivalent to the relation

$$\sum_{\alpha}\int d\mathbf{p}_{\alpha}\varphi_{\alpha}f_{\alpha}{}^{\circ}\left[v_{\alpha z}\frac{\partial \ln f_{\alpha}{}^{\circ}}{\partial z}+v_{\alpha x}\frac{\partial \varphi_{\alpha}}{\partial x}-I\left(\varphi_{\alpha}\right)\right]=0 \qquad (3.2)$$

The exact solution of Eq. (1.4) satisfies also equality (3.2). We stipulate that the approximate solution is to satisfy also (3.2). Let us consider a simple gas. We rewrite equality (3.2) in the form

$$\int d\mathbf{p}\varphi f^{\circ}\left[v_{z}\frac{\partial \ln f^{\circ}}{\partial z}+v_{x}\frac{\partial \varphi}{\partial x}\right]=\int d\mathbf{p}\varphi I\left(\varphi\right) \qquad (3.3)$$

and seek the solution of Eq. (1.4) in the form of a series in certain polynomials of molecule velocity $\mathbf{c}$

$$\varphi=\sum_{i}a_{i}\left(x\right)P_{i}\left(\mathbf{c}\right), \quad \mathbf{c}=\mathbf{v}\Big/\sqrt{\frac{2kT}{m}}$$

Substituting φ into the left-hand side of equality (3.3) we obtain

$$\int d\mathbf{p}\sum_{i}a_{i}\left(x\right)f^{\circ}P_{i}\left(\mathbf{c}\right)\left[v_{z}\frac{\partial \ln f^{\circ}}{\partial z}+v_{x}\sum_{k}\frac{\partial a_{k}\left(x\right)}{\partial x}P_{k}\left(\mathbf{c}\right)\right]=\sum_{i}a_{i}J_{i} \qquad (3.4)$$

$$J_{i}=\int d\mathbf{p}f^{\circ}P_{i}\left(\mathbf{c}\right)\left[v_{z}\frac{\partial \ln f^{\circ}}{\partial z}+v_{x}\sum_{k}\frac{\partial a_{k}\left(x\right)}{dx}P_{k}\left(\mathbf{c}\right)\right]$$

i.e. the local entropy production is also represented in the form of a bilineal combination of forces a_i and fluxes J_i. Hence the local thermodynamic equations of motion can also be represented in the form

$$J_{i}=\sum_{l}L_{il}a_{l} \qquad (3.5)$$

with coefficients L_{il} satisfying the Onsager symmetry relation. The comparison of (3.4) and (3.5) shows that (3.5) are moment equations derived from Eq. (1.4) with the use of the same polynomials that were used for approximating function φ, and that coefficients L_{il}

$$L_{il}=\int f^{\circ}P_{i}I\left(P_{l}\right)d\mathbf{p}$$

This method of deriving moment equations seems to be preferable to the conventional formal one in which arbitrary velocity polynomials are used, since it a priori ensures the fulfilment of the entropy balance condition. This with the use of equality (3.3) makes it possible to repeat the reasoning in Sect. 2 also for the approximate solution of $\varphi\left(x, \mathbf{c}\right)$, and obtain Eqs. (2.5) and (2.6) with coefficients that satisfy the Onsager principle.

4. Let us apply the indicated method for determining function $\varphi\left(x, \mathbf{c}\right)$ and the macro-properties of the flow of gas in a plane channel in which small independent longitudinal pressure and temperature gradients are maintained. We shall use the method of half-space expansions [3,4]. Following Maxwell we assume that the molecule distribution function becomes discontinuous when $v_x = 0$ in proximity of the channel

walls. However we specify that within the volume of gas it must be continuous and converts to the known Chapman - Enskog solution. These conditions determine the form of approximation of function $\varphi(x, \mathbf{c})$

$$\varphi^{\pm} = c_z \, [a_0^{\pm}(x) + c_x a_1^{\pm}(x) + (c^2 - {}^5/_2)\, a_2^{\pm}(x) + (c_x^2 - {}^1/_2)\, a_3^{\pm}(x)]$$

For the boundary conditions we use Maxwell's condition: portion ε of molecules is diffusely reflected from the walls, while the reflection of the $1 - \varepsilon$ portion is specular. We omit solution details (see Sect.5) and present the obtained formulas for mean velocity and densities of momentum and heat fluxes

$$u_z(x) = \frac{1}{2\eta}\left[(x^2 - L^2) - \frac{5\pi}{8}\lambda L \frac{2-\varepsilon}{\varepsilon} F_1(x, \varepsilon)\right]\nabla_z p - \frac{15}{18}\frac{\eta}{\rho} F_2(x, \varepsilon)\nabla_z \ln p + \frac{3}{4}\frac{\eta}{\rho} F_3(x, \varepsilon)\nabla_z \ln T \tag{4.1}$$

$$\sigma_{xz} = -m\int d\mathbf{p} v_x v_z \varphi(x) f^{\circ} = x\nabla_z p \tag{4.2}$$

$$q_z(x) = \frac{3}{2}\frac{\eta}{\rho}\left[1 + \frac{1}{2}\Phi_1(x, \varepsilon) + \frac{\varepsilon}{2-\varepsilon}\,\mathrm{Kn}_2\Phi_2(x, \varepsilon)\right]\nabla_z p - \varkappa\left[1 + \frac{\varepsilon}{2-\varepsilon}\,\mathrm{Kn}_2\,\Phi_3(x, \varepsilon)\right]\nabla_z T \tag{4.3}$$

where $\varkappa$ and η are the coefficients of thermal conductivity and dynamic viscosity of gas. In the derivation of these formulas it was assumed that the gas is monatomic, molecules were simulated by hard spheres, and functions $F_i(x, \varepsilon)$ and $\Phi_i(x, \varepsilon)$ were those defined in Sect.5.

Let us consider equality (4.1). Its first two terms were analyzed in some detail in [1], where a similar formula was obtained by the method of elementary solutions with the use of the Batnagar - Gross - Crook model. The third term defines the thermal slip phenomenon; it consists of three components:

1) a quantity formally independent (for $\varepsilon \neq 0$) on the law of interaction between gas molecules and wall; this part of thermal slip was determined in the known work of Maxwell;

2) slip not accounted for by Maxwell which is due to the distortion of the distribution function of molecules impinging on the wall as the result of their interaction with molecules reflected by the wall;

3) the related to this effect of the appearance of the thermal slip velocity profile localized in the Knudsen layer. Because of this the micro- and macroscopic thermal slip velocities differ.

The similarity structure of the second and third terms of the right-hand side of equality (4.1) suggests the possibility of considering the first of these as the slip induced by a pressure gradient. We shall call it "baroslip" (in [1] this effect is called second order slip).

According to the principle of duality an isothermal heat flux defined in the first set of brackets in (4.3) corresponds to the thermal slip. Its component which is independent of the coordinate, as shown in the book by Chapman and Cowling, is the so-called bulk isothermal transport heat. The second part of that heat is localized in the Knudsen layer. It is also dependent on the law of interaction between gas molecules and wall. The nature of heatconduction is similar: its volume component is defined by the usual thermal conductivity coefficient and the correction defines heat transfer in the boundary layer.

We average (4.1) and (4.3) over the channel cross section and compare the result with Eqs. (2.5). We can then write formulas for kinetic coefficients L_{ik} the expressions

$$L_{11} = -\varkappa T^2 \left[1 + \frac{\varepsilon}{2-\varepsilon} \mathrm{Kn}_2 \overline{\Phi}_3(\varepsilon, \mathrm{Kn}_2)\right]$$

$$L_{12} = \frac{3}{2}\frac{\eta}{\rho} T \left[1 + \frac{1}{2}\overline{\Phi}_1(\varepsilon, \mathrm{Kn}_2) + \frac{\varepsilon}{2-\varepsilon} \mathrm{Kn}_2 \overline{\Phi}_2(\varepsilon, \mathrm{Kn}_2)\right]$$

$$L_{21} = \frac{3}{4}\frac{\eta}{\rho} T \overline{F}_3(\varepsilon, \mathrm{Kn}_2)$$

$$L_{22} = -\frac{T}{2\eta}\left[\frac{2}{3}L^2 + \frac{5\pi}{8}L^2 \mathrm{Kn}_2 \frac{2-\varepsilon}{\varepsilon}\overline{F}_1(\varepsilon, \mathrm{Kn}_2) + \frac{5}{9p}\frac{\eta}{\rho}\overline{F}_2(\varepsilon, \mathrm{Kn}_2)\right]$$

where the upper dash denotes values of related functions averaged over the channel cross section. According to the principle of Onsager $L_{12} = L_{21}$, i.e. the equality

$$\frac{\overline{F}_3}{2} = 1 + \frac{\overline{\Phi}_1}{2} + \frac{\varepsilon}{2-\varepsilon}\mathrm{Kn}_2\overline{\Phi}_2 \tag{4.4}$$

must be satisfied.

Calculations (see Sect. 5) had confirmed that the derived approximate solution satisfies that equality (in the limit case of $\mathrm{Kn}_2 \gg 1$ this result is obtained analytically, however, the computation of coefficients was generally effected by numerical methods).

5. Appendix. A detailed description of the procedure for determining the coefficients of the expansion of function $\varphi(x, c)$ approximated by three velocity polynomials, appears in [4]. The method used in this paper does not differ from that, except for the number of polynomials used in the approximation. We use the notation

$$x_{1,2} = a_0^+ \pm a_0^-, \quad x_{3,4} = a_1^+ \pm a_1^-, \quad x_{5,6} = a_2^+ \pm a_2^-, \quad x_{7,8} = a_3^+ \pm a_3$$

For the derivation of moments of the kinetic equation we use polynomials

$$c_z, \quad c_x c_z, \quad c_z\left(c^2 - \frac{5}{2}\right), \quad c_z\left(c_x^2 - \frac{1}{2}\right)$$

$$c_z \frac{c_x}{|c_x|}, \quad c_z \frac{c_x^2}{|c_x|}, \quad c_z\left(c^2 - \frac{5}{2}\right)\frac{c_x}{|c_x|}, \quad c_z\left(c_x^2 - \frac{1}{2}\right)\frac{c_x}{|c_x|}$$

The solution of the system of linear differential equations yields for $x_i(x)$ the formula

$$x_i(x) = 2\sum_{m,\,j=1}^{3} \frac{1}{|B_{ij}|}\gamma_{ij}(-1)^{j+m} A_m C_{mj} \times \begin{cases} \mathrm{sh}\,\alpha_j x, & i = 3, 6, 8 \\ \mathrm{ch}\,\alpha_j x & i = 4, 5, 7 \end{cases}$$

where $|B_{ij}|$ is the determinant with elements

$$B_{1j} = \gamma_{3j}\,\mathrm{sh}\,\alpha_j L - \frac{\varepsilon}{2-\varepsilon}\gamma_{4j}\,\mathrm{ch}\,\alpha_j L$$

$$B_{2j} = \gamma_{6j}\,\mathrm{sh}\,\alpha_j L - \frac{\varepsilon}{2-\varepsilon}\gamma_{5j}\,\mathrm{ch}\,\alpha_j L$$

$$B_{3j} = \gamma_{8j}\,\mathrm{sh}\,\alpha_j L - \frac{\varepsilon}{2-\varepsilon}\gamma_{7j}\,\mathrm{ch}\,\alpha_j L, \quad j = 1, 2, 3$$

where C_{mj} is a minor of determinant $|B_{ij}|$ obtained by the deletion of the m-th row and j-th column.

We adduce for reference, the values of constants γ_{ij} $(\gamma_{5j} = 1)$

$\gamma_{31} = 11.751$, $\gamma_{41} = -19.4759$, $\gamma_{61} = -0.4344$, $\gamma_{71} = 13.779$
$\gamma_{81} = -9{,}0647$
$\gamma_{32} = -3.1047$, $\gamma_{42} = 3{,}5278$, $\gamma_{62} = -0{,}7640$, $\gamma_{72} = 2.5864$
$\gamma_{82} = 2.1941$
$\gamma_{33} = 19.5218$, $\gamma_{43} = -25.7145$, $\gamma_{63} = -0.6748$, $\gamma_{73} = 8.7753$
$\gamma_{83} = -6.5084$

The eigenvalues are

$$\alpha_1 = 0.80307\,/\,\lambda, \quad \alpha_2 = 1.5967\,/\,\lambda, \quad \alpha_3 = 3.7789\,/\,\lambda$$

Symbol A_m denotes the combinations

$$A_1 = 2L\,\nabla_z \ln p$$

$$A_2 = \frac{\varepsilon}{2-\varepsilon}\cdot\frac{3}{4}\frac{\eta}{\rho}\cdot\frac{8}{\sqrt{\pi}\,\bar{v}}\left(\frac{2}{9}\nabla_z \ln p - \nabla_z \ln T\right)$$

$$A_3 = -\frac{\varepsilon}{2-\varepsilon}\cdot\frac{3}{4}\frac{\eta}{\rho}\cdot\frac{8}{\sqrt{\pi}\,\bar{v}}\cdot\frac{8}{9}\nabla_z \ln p$$

The velocity of gas and the heat flux are

$$u_z(x) = \frac{\sqrt{\pi}}{8}\bar{v}\left[x_1(x) + \frac{1}{\sqrt{\pi}}x_4(x)\right]$$

$$q_z(x) = \frac{p}{8}\left(\frac{2kT}{m}\right)^{1/2}\left[5x_5(x) + \frac{1}{\sqrt{\pi}}x_4(x) + x_7(x)\right]$$

The substitution of the expressions for $x_i(x)$ yields

$$u_z(x) = \frac{\sqrt{\pi}}{8}\bar{v}\left\{\frac{\sqrt{\pi}}{2}\frac{\rho\bar{v}}{\eta}(x^2 - L^2)\nabla_z \ln p - \frac{2-\varepsilon}{\varepsilon}(\sqrt{\pi}A_1 + A_2 + A_3) + \right.$$
$$\frac{1}{|B_{ij}|}\sum_{m,\,j=1}^{3}(-1)^{j+m}A_m C_{mj}\left[\left(\frac{4-\pi}{\sqrt{\pi}}\gamma_{4j} + 1 + \gamma_{7j}\right)\mathrm{ch}\,\alpha_j L - \right.$$
$$\left.\left.2\left(\frac{1}{\sqrt{\pi}}\gamma_{4j} + 1 + \gamma_{7j}\right)\mathrm{ch}\,\alpha_j x\right]\right\}$$

$$q_z(x) = \frac{p}{4}\left(\frac{2kT}{m}\right)^{1/2}\left\{\frac{2-\varepsilon}{\varepsilon}(5A_2 + A_3) + \right.$$
$$\left.\frac{1}{|B_{ij}|}\sum_{m,\,j=1}^{3}(-1)^{m+j}A_m C_{mj}\left(5 + \frac{1}{\sqrt{\pi}}\gamma_{4j} + \gamma_{7j}\right)\mathrm{ch}\,\alpha_j x\right\}$$

The introduction of notation

$$D_{1j} = C_{1j}, \quad D_{2j} = -{}^{1}/_{5}\,(C_{2j} - 4C_{3j}), \quad D_{3j} = -C_{2j}$$

$$G_{1j} = \frac{2}{\sqrt{\pi}}C_{1j}, \quad G_{2j} = -\frac{5}{48}(C_{2j} - 4C_{3j}), \quad G_{3j} = -\frac{3}{16}C_{2j}$$

$$M_j = -2\left(\frac{1}{\sqrt{\pi}}\ \gamma_{4j} + 1 + \gamma_{7j}\right)\left(\frac{4-\pi}{\sqrt{\pi}}\gamma_{4j} + 1 + \gamma_{7j}\right)^{-1}$$

$$F_i = 1 + \frac{\varepsilon}{2-\varepsilon}\sum_j (-1)^j \frac{D_{ij}}{|B_{ij}|}\left(\frac{4-\pi}{\sqrt{\pi}}\ \gamma_{4j} + 1 + \gamma_{7j}\right) \times$$

$$\operatorname{ch}\alpha_j L\left(1 + M_j\frac{\operatorname{ch}\alpha_j x}{\operatorname{ch}\alpha_j L}\right)$$

$$\Phi_i = L\sum_j (-1)^j \frac{G_{ij}}{|B_{ij}|}\left(\frac{\sqrt{\pi}}{2}\ \gamma_{3j} + 6\gamma_{6j} + 2\gamma_{8j}\right)\alpha_j \operatorname{ch}\alpha_j x$$

yields formulas (4.1) and (4.3).

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